Sum-to-Product

$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right) \times \cos\left(\frac{u-v}{2}\right)$

$\sin u - \sin v = 2\sin\left(\frac{u-v}{2}\right) \times \cos\left(\frac{u+v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right) \times \sin\left(\frac{a-v}{2}\right)$

$\cos u + \cos v = -2\cos\left(\frac{u+v}{2}\right) \times \cos\left(\frac{a-v}{2}\right)$

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Product-to-Sum

$\cos\alpha \times \cos\beta = \frac{1}{2} \times [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha \times \cos\beta = \frac{1}{2}\,[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\sin\alpha \times \sin\beta = \frac{1}{2}\,[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha \times \sin\beta = \frac{1}{2}\,[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

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$\frac{\sin(13w) + \sin(3w)}{\cos(13w) + \cos(3w)} = \tan(8w)$

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$\frac{\sqrt{3}+1}{\sqrt{3}-1}$ is ultimately simplified.

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$\sin\theta = cos 90 - \theta$

$\tan\theta = cot 90 - \theta$

$\sec\theta = \csc 90 - \theta$

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When simplifying a denominator with two terms, multiply by the conjugate.

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Half-Angle Identities; Product-to-Sum Identities:

$\cos\left(\frac{\theta}{2}\right) = \cos\left(\frac{1}{2}\,\theta\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}.$

$\sin\left(\frac{\theta}{2}\right) = \sin\left(\frac{1}{2}\,\theta\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}.$

$\tan\left(\frac{\theta}{2}\right) = \tan\left(\frac{1}{2}\,\theta\right) = \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}.}$

$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

$\cot\left(\frac{\theta}{2}\right) = \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}}.$

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Double-Angle Identities:

cos (2a) = cos (a + a) = cos²a - sin²a = 2cos²(a) - 1 = 1 - 2sin²(a).

sin (2a) = 2sin(a) × cos(a).

$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}.$

$\cot(2\theta) = \frac{1-\tan^2\theta}{2\tan\theta}.$

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When finding an angle not on the Unit Circle, subtraction or additions of values on the unit circle works for multiples of 5°.

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Sum Identities

$\cos(a+b) = \cos(a) \times \cos(b) - \sin(a) \times \sin(b)$

$\cos(a-b) = \cos(a) \times \cos(b) + \sin(a) \times \sin(b)$

$\sin(a+b) = \sin(a) \times \cos(b) + \cos(a) \times \sin(b)$

$\sin(a-b) = \sin(a) \times \cos(b) - \cos(a) \times \sin(b)$

$\tan(a+b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a) \times \tan(b)}$

$\tan(a-b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a) \times \tan(b)}$

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If f(-x) = -f(x), then f() is an odd function. sin is an odd function. csc is too. cos is not. tan is an odd function. If f(-x) = f(x), then f() is an even function. Cos is an even function.

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Pythagorean identities:

$\sin^2 + \cos^2 = 1$, $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \csc^2\theta$.

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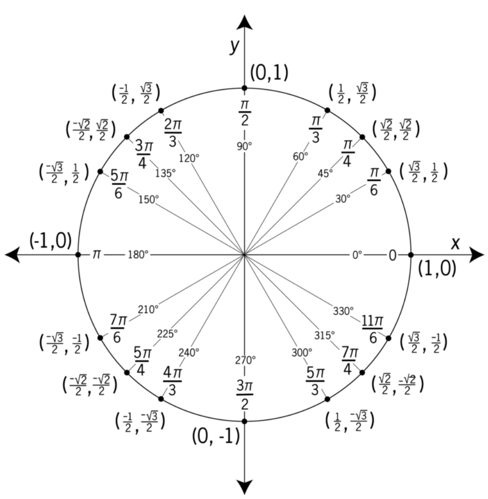


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$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$, $\csc(-\theta) = -\csc\theta$, $\sec(-\theta) = \sec\theta$, $\cot(-\theta) = -\cot\theta$.

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Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4 \times a \times c}}{2 \times a}$

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Solve the equation (x in radians and θ in degrees) for all exact solutions where appropriate. Round approximate answers in radians to four decimal places and approximate answers in degrees to the nearest tenth. $\cos^2 x - \cos x = 0$ ¶ Cosine returns 0 at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Being asked to find all exact solutions, a rule may be expressed. $\frac{\pi}{2} + \pi \times n$. When cosine returns 1, the equation given simplifies as follows: $1^2 - 1 = 0$. Cosine returns 1 at 0 and 2π. This may be expressed by the rule $2 \times \pi \times n$.

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$\sin 2\theta = -\frac{1}{2}$ ¶

$2\theta = \sin^{-1}\left(\frac{-1}{2}\right)$

$2\theta = -30°$

$\theta = -15° = 345°$

$\sin 2\theta$ has a period of π. Two angles in the same quadrant.

$270° + (360° - 345°) = 285°$

To get inverse angles, subtract 180° from each anwer. $285° - 180° = 105°$. $345° - 180° = 165°$.

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$\sin(2x) + \sin(x) = 0.$ ¶

$\sin 2x = -\sin x$

This is true when sine returns 0.

$0, \pi$

$2\sin x \times \cos x + \sin x = 0$

$\sin x \times (2\cos x + 1) = 0$

$\sin x = 0$

As previously found, true at 0 and π.

$2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

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So it was written.